

A Numerical Study of Plume Separation above a Thin Fin in a Differentially Heated Cavity

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Abstract

The separation of plumes from a horizontal fin attached to a sidewall of a differentially heated cavity is numerically studied over a range of Rayleigh numbers and fin heights. The simulation results reveal that the plume separation frequency increases with the Rayleigh number and decreases with the fin height. The decrease of the separation frequency with the fin height is mainly due to the cavity scale temperature stratification. It is demonstrated that the temperature difference in the unstable layer above the fin decreases with increasing fin height. Both the local Reynolds number and Rayleigh number in the unstable layer are non-monotonic functions of the fin height due to the complex flow patterns in the cavity caused by the fin. It is also found that the heat transfer rate through the heated sidewall is significantly enhanced due to temperature oscillations induced by the separating plumes. An optimum fin height for heat transfer enhancement has been identified for Rayleigh numbers greater than 1.84×10^9 .

Introduction

Natural convection in a differentially heated cavity is a classical heat transfer problem which was first studied in the 1950s [1]. Because of its significance for both fundamental fluid mechanics and industrial applications, this problem has drawn much attention since then [2].

In the literature, many methods designed to enhance the heat transfer across the cavity have been reported, for example by imposing a sinusoidally-varying temperature wall condition [3] or by including a rotating cylinder in the cavity [4]. In recent years the transient flow associated with a horizontally attached adiabatic thin fin on the cavity sidewall was intensively investigated [5-7]. This is a passive method for heat transfer enhancement. It is demonstrated that by the presence of the thin fin, the flow above the fin is triggered to be unstable and some chaotic characteristics can be observed downstream of the fin [6] where the heat transfer is enhanced by up to 23% at the early stage and about 5% at the quasi-steady stage. It is believed that a Rayleigh-Benard type instability in a form of separating plumes from the fin is the cause of the unstable flow downstream. Thus, understanding the flow above the fin, especially the plume separation frequency and its dependency on other flow parameters, is of great importance. This is the motivation of the present work.

In this paper, natural convection in a differentially heated finned cavity is numerically studied over a range of Rayleigh numbers and fin heights. We find that the plume separation frequency depends on both the Rayleigh number and fin height. Since the fin triggers periodic plume separation which in turn affects the downstream flow, the heat transfer there is significantly enhanced by the strong oscillations induced by the fin.

Physical Problem

The problem considered in the present study is a differentially heated cavity with a horizontal adiabatic thin fin attached to the heated sidewall. The ceiling and bottom of the cavity are adiabatic.

The initial fluid temperature in the cavity is T_0 and the initial velocity is set to zero (i.e. isothermal and stationary). The temperature of heated sidewall and cold sidewall is $T_h = T_0 + \Delta T/2$ and $T_c = T_0 - \Delta T/2$, respectively. It has been demonstrated that the flow in the differentially heated cavity can be well characterized by a two-dimensional (2D) numerical model as suggested in [6]. Therefore, we only consider the flow in 2D. The natural convection flow in the cavity can be characterised by four governing parameters: the Rayleigh number (Ra), the Prandtl number (Pr), the cavity aspect ratio (A) and the dimensionless fin height (h/H). They are defined as follows:

$$Ra = \frac{g\beta\Delta TH^3}{\nu\kappa}, \quad Pr = \frac{\nu}{\kappa}, \quad A = \frac{H}{L}, \quad \phi = \frac{h}{H}.$$

where g , β , ν , k , H , L and h are gravitational acceleration, thermal expansion coefficient, kinematic viscosity, thermal diffusivity, height and length of the cavity, and the dimensional fin height. The aspect ratio of the cavity studied here is 0.24, and the fin length is 1/6, nondimensionalized using the cavity height H . These dimensions are adopted based on the experimental model used in [5]. The cavity is filled with water with a fixed Prandtl number of 6.64. Four different Rayleigh numbers ($Ra = 0.92 \times 10^9$, 1.84×10^9 , 3.68×10^9 and 7.36×10^9) and seven different fin heights (i.e. 1/4, 1/3, 5/12, 1/2, 7/12, 2/3, 3/4) are studied to investigate the dependence of the plume separation frequency on the Rayleigh number and fin height. The schematic of the computational domain with a thin fin at the mid-height is shown in figure 1.

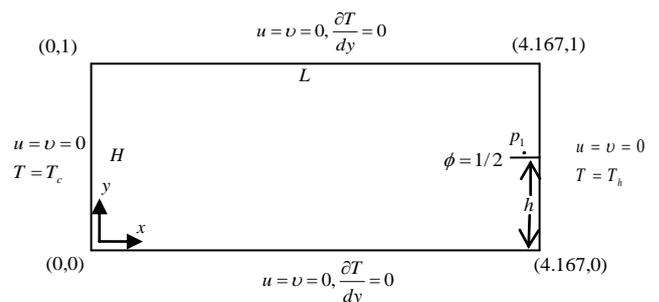


Figure 1. Schematic of the computational domain and monitoring point location: p_1 at (4.042, 0.5167).

Numerical Procedures

A two-dimensional model is employed to account for the current natural convection flow problem. The governing equations can be expressed in the following non-dimensional form after the variables are non-dimensionalized using the following scales: $x \sim H$; $t \sim H^2/\nu$; $(T-T_c) \sim (T_h-T_c)$; $u, v \sim \nu/H$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{Ra\Delta T}{Pr} \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

where u and v are the velocity components in x and y directions respectively, and ρ, t, p, T , are the density, time, pressure and temperature, respectively. The above governing equations are subject to the following boundary conditions: the cold sidewall is maintained at a low temperature T_c and the heated sidewall at a high temperature T_h . The adiabatic boundary condition is imposed at the ceiling and bottom. All surfaces are no-slip. The momentum and energy equations are discretized using the QUICK scheme. A second order implicit scheme is employed for the transient formulation. The SIMPLE algorithm is applied for the pressure-velocity coupling. The governing equations are solved iteratively and the normalised convergence criteria – the residuals are set to 10^{-6} for the energy equation and 10^{-3} for all the other equations.

A structured grid with 520×280 cells is established for the computational domain with finer mesh in the wall and fin vicinities and a time step of 8.24×10^{-7} is employed. The grid system and time step are determined according to the mesh and time step independency tests reported in [6, 8].

Results and discussion

Of great interest is the fin downstream flow, and the transient development of this flow can be roughly classified into three stages, i.e. early stage, transitional stage and quasi-steady (periodic) stage [5]. The current work will only focus on the plume separation in the quasi-steady stage and the first two stages are thus not discussed herein.

As is known for a differentially heated cavity without a fin, hot fluid is detrained from the upper part of the heated sidewall into the cavity, while cold fluid is entrained from the lower part of the cavity, forming cavity-scale stratification in the quasi-steady stage. With the presence of a horizontal fin, the upstream boundary layer flow wraps around the fin at quasi-steady stage (refer to figure 2a), during which the temperature structure in the upstream thermal boundary layer is approximately preserved. Accordingly, for the thermal flow formed above the fin, the temperature is higher towards the fin surface and lower away from the fin. Therefore, the vertical temperature profile between the upper surface of the fin and the ceiling consists of two layers as shown in figure 2b, an upper layer with stable stratification and a lower layer with an adverse temperature gradient. The thermal flow above the fin is not stable in the Rayleigh–Benard sense and will lead to the plume separation motions above the fin.

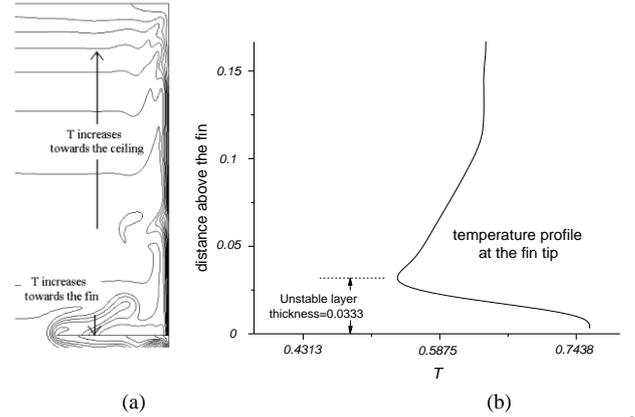


Figure 2. Unstable thermal layer structure above the fin ($Ra=1.84 \times 10^9$ and $\phi=1/2$).

It is demonstrated experimentally [7] that there are no distinct temperature oscillations in the thermal boundary layer upstream of the thin fin. Oscillations can only be observed in the boundary layer after the flow bypasses the fin tip, and they are associated with the plume separations. The temperature time series obtained at point p1 on the top side of the fin at the quasi-steady stage is shown in figure 3, which confirms that the temperature is indeed varying periodically with time. Accordingly, it numerically confirms the existence of intermittent plume separations.

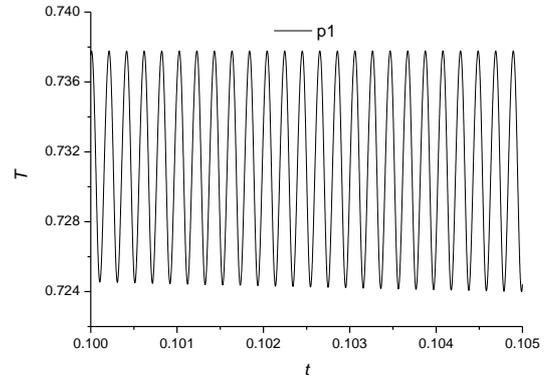


Figure 3. Periodic temperature oscillation above the fin ($Ra=1.84 \times 10^9$ and $\phi=1/2$).

The plume separates periodically at a constant time interval and the separation frequency can be obtained by performing a FFT (Fast Fourier Transform). The results for different fin heights and Rayleigh numbers are given in figure 4. It can be seen in this figure that for the same fin height, the separation frequency increases with the Rayleigh number. This is because the flow is more unstable at higher Rayleigh numbers. It is also seen in figure 4 that the separation frequency generally decreases as the fin height increases for the same Rayleigh number. This is mainly caused by different temperature structures of the unstable layer above the thin fin, which is described below.

Due to the cavity scale temperature stratification, the fin ambient temperature is higher towards the cavity ceiling at the quasi-steady stage. This leads to a smaller temperature difference in the unstable layer for higher fin position, and thus the unstable layer above the fin becomes less unstable. Figure 5a replots the plume separation frequency along with the temperature difference across the unstable layer above the fin against the fin position. A clear correlation between the plume separation frequency and the temperature difference across the unstable layer can be seen in figure 5a.

The unstable flow above the fin is similar to the Poiseuille-Rayleigh-Benard flow. For such a flow configuration, the inflow condition is also important for the unstable layer flow besides the local Rayleigh number. The results of maximum horizontal flow velocity u_{max} , the unstable layer thickness δ_{layer} , the local Reynolds number Re_l and the local Rayleigh number Ra_l , as defined by equation 5 below, are presented in figures 5b and 5c.

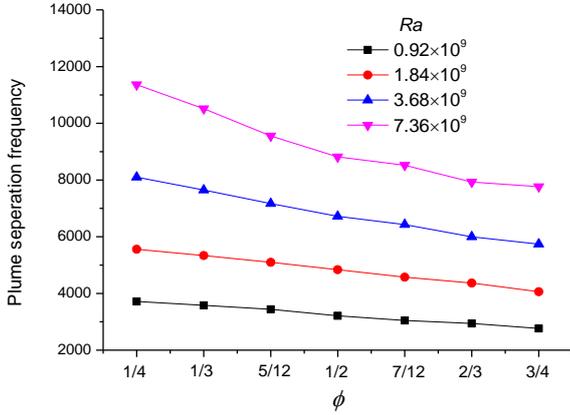


Figure 4. Plume separation frequency results of different ϕ and Ra .

$$Re_l = \frac{\delta_{layer} u_{max}}{\nu} \quad (5a)$$

$$Ra_l = \frac{g\beta(T_{layer,high} - T_{layer,low})\delta_{layer}^3}{\nu\kappa} \quad (5b)$$

It can be seen in figure 5b that u_{max} is not a monotonic function of the fin height. With the increasing fin height, it first increases and then decreases, resulting in a similar behaviour of the local Reynolds number. This is explained below based on the flow velocity patterns in the finned cavity.

Natural convection in a differentially heated cavity without a fin is totally symmetric about the cavity centre, meaning that fluid entrainment happens below the cavity mid-height plane whilst detrainment happens above that plane for the heated sidewall boundary layer. In the case with a fin placed at a position higher than the mid-height, the entrainment process is quite similar to the non-fin cavity case from the cavity bottom to mid-height. The detrainment process occurs from above the mid-height plane to a certain distance below the fin, followed by entrainment of fluid by the boundary layer downstream of the fin. The fin downstream boundary layer also detrains fluid for the cold sidewall boundary layer development. The final result is a three velocity cell pattern as shown in figure 6. Note that all the velocity vector indicators are presented in the same scale to facilitate observation. The detrained mass flux from the upstream boundary layer decreases the mass flux arriving at the fin tip and in turn reduces the through flow velocity and local Reynolds number above the fin.

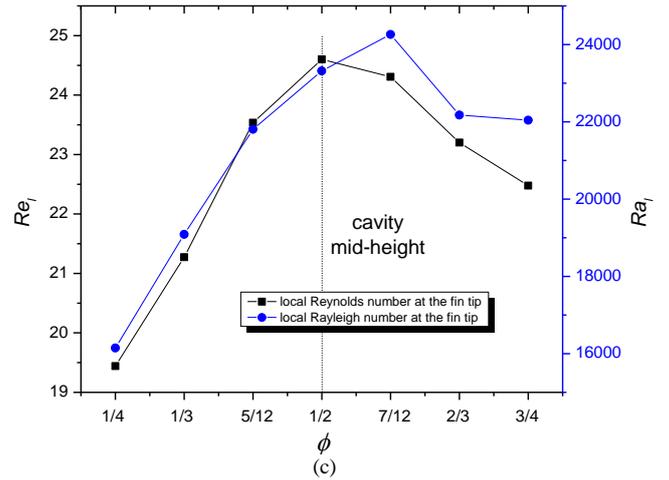
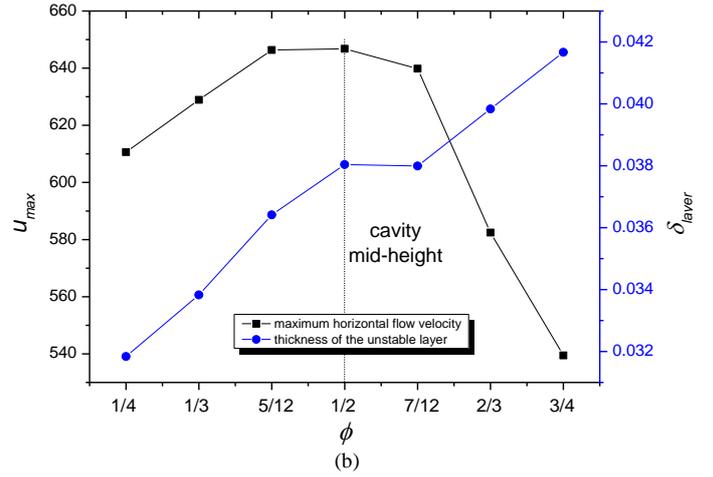
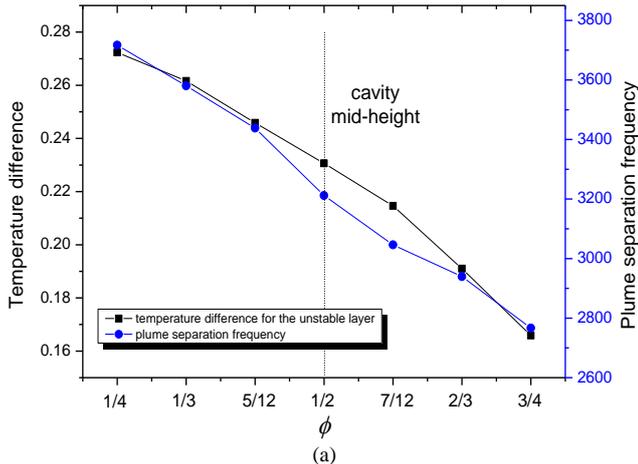


Figure 5. Local parameters for the unstable layer ($Ra=0.92 \times 10^9$).

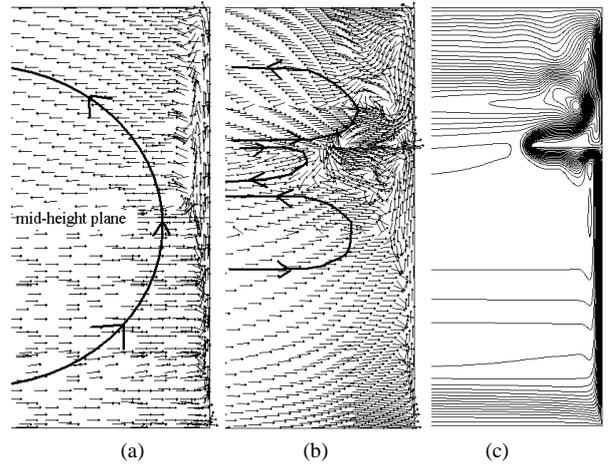


Figure 6. Flow velocity pattern (a) velocity vectors for cavity without a fin. (b) velocity vectors for $\phi=2/3$ case. (c) streamline for $\phi=2/3$ case. ($Ra=0.92 \times 10^9$).

Heat transfer enhancement

The appearance of the intermittent plume separation acts as a strong perturbation for the fin downstream flow where some turbulence characteristics can be observed. This in turn enhances the downstream heat transfer. To quantitatively assess the heat transfer enhancement, the Nusselt number along the sidewall is integrated and an enhancement factor is defined as:

$$\varepsilon = \frac{Nu_{fin} - Nu_{nofin}}{Nu_{nofin}} \quad (6)$$

The calculated heat transfer enhancement factor for $Ra = 1.84 \times 10^9$ at various fin positions is plotted in figure 7. It is seen in this figure that heat transfer is enhanced by the presence of the fin in all the cases. In the early stage, the enhancement factor reaches as high as 25%, depending on the fin position. With the passage of time, it gradually drops to about 5.6% at time 0.11 for the $\phi = 1/2$ case.

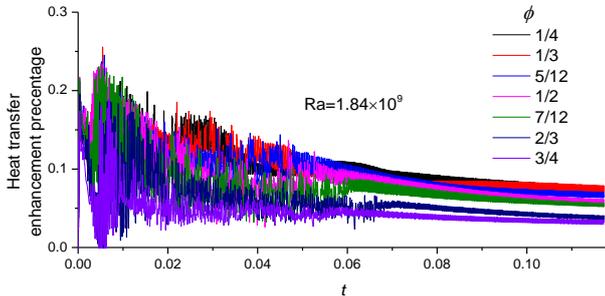


Figure 7. Heat transfer enhancement percentage of different fin heights.

It is also clearly seen from figure 7 that ε is also oscillating in the quasi-steady stage because of the plume motions. The averaged Nusselt number of the heated sidewall at the quasi-steady stage is obtained and presented in figure 8. It can be seen that there is an optimum fin height for heat transfer enhancement purpose. For cases with Rayleigh number greater than 1.84×10^9 , the optimum fin height is $1/3$. However, the heat transfer enhancement factor decreases monotonically with the increasing fin height for the 0.92×10^9 case. The cause of this inconsistent behaviour with the other cases is unclear, and further studies are being carried out to resolve the matter.

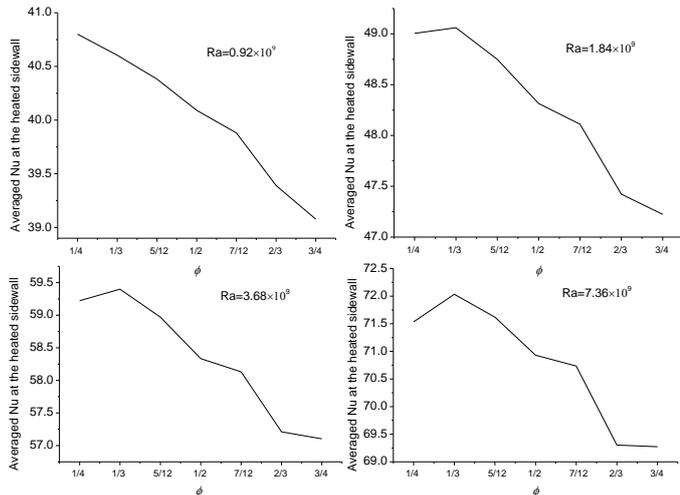


Figure 8. Averaged Nusselt number at quasi-steady stage.

Conclusions

Fin induced Rayleigh-Benard type instabilities above a fin in a differentially heated cavity are numerically studied in this paper.

The results suggest that temperature oscillations are caused by the unstable layer above the fin and intermittent plumes are separated from this unstable layer. The present results demonstrate that the plume separation frequency increases with increasing global Rayleigh number and decreasing fin height, whilst the local maximum horizontal velocity, Reynolds and Rayleigh numbers of the unstable flow exhibit non-monotonic behaviours with the fin position due to the complex flow patterns caused by the fin.

We also find that the heat transfer rate through the heated sidewall is significantly enhanced by the temperature oscillations induced by the separated plumes. For Rayleigh numbers greater than 1.84×10^9 , one third of the cavity height is found to be the optimum fin position for heat transfer enhancement purpose among all the fin heights investigated in this study.

Acknowledgments

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